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Deterministic Clone of an Unknown N-Qubit Entangled State with Assistance

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Abstract We propose a deterministic scheme to realize assisted-clone of an unknown $N(\geq 3)$-qubit entangled state. The first stage of the protocol requires teleportation via maximal entanglement as the quantum channel. In the second stage of the protocol, a novel set of mutually orthogonal basis vectors are constructed. With the assistance of the preparer through an N-particle projective measurement under this basis, the perfect copy of an original state can be produced. Comparing with the previous protocols which produce the unknown state and its orthogonal complement state at the site of the sender, our scheme generates the unknown state deterministically.

Keywords Deterministic assisted-clone · Complete orthogonal basis · Projective measurement · Maximal entanglement

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1 Introduction

An unknown quantum state cannot be cloned exactly. It follows from the celebrated no-cloning theorem [1]. However, quantum cloning approximately is necessary in quantum information [2] and is also applicable to other interesting quantum processes, e.g., quantum network coding [3], etc. Hence, in literatures various cloning machines [4–9] have been proposed which operate either in a deterministic or probabilistic way.

In 2000, Pati [10] proposed a scheme for cloning an unknown single-qubit state with minimal assistance from a state preparer. The perfect copies and orthogonal-complement copies of an unknown state can be produced. Inspired by Pati's original paper, a lot of schemes for assisted cloning have been proposed [11–23]. Among them, some protocols concentrated on the clone of an unknown $N(\geq 3)$-qubit entangled state

$$|\varphi\rangle_{1\cdots N} = \alpha_0|0\cdots 0\rangle_{1\cdots N} + \alpha_1|1\cdots 1\rangle_{1\cdots N}, \tag{1}$$

where α_0 is a real number, α_1 is a complex number and $|\alpha_0|^2 + |\alpha_1|^2 = 1$. Specially, Shi et al. [22] proposed a scheme for $N = 3$. Ma et al. [23] investigated the general case with $N \geq 3$. However, these protocols [22, 23] have some features in common: One is that Bell measurements are used by the sender to achieve teleportation [24, 25]. The other is that the preparer carries out an N-particle projective measurement on his particles sent by the sender directly after the teleportation. Then the unknown state can be produced at the site of the sender with the successful probability $\frac{1}{2}$, which means these protocols cannot always generate the copy of the unknown state deterministically.

In this paper, we reinvestigate the assisted-clone of an unknown $N(\geq 3)$-qubit entangled state in (1). We construct a unitary transformation and a set of appropriate basis vectors which play an important role in our scheme. In detail, after teleportation [24] the sender first performs this unitary transformation on her particles, and sends parts of the transformed particles to the preparer. Then the preparer performs an N-particle projective measurement under this basis. By rigorous formulations, it is proved that the sender can reestablish the unknown state deterministically while the cost of quantum nonlocal resource is the same as its in [22, 23]. Comparing with the previous protocols [22, 23], which produce the unknown state and its orthogonal complement state at the site of the sender, our scheme generates the unknown state deterministically.

2 Deterministic Clone of an Unknown Tri-Qubit Entangled State with Assistance

To be explicit, we first consider the case of tri-qubit entangled state (i.e., with $N = 3$) and then deal with that of arbitrary $N(\geq 3)$-qubit entangled state.

Suppose there are three participants, Victor, Alice and Bob. Victor is the state preparer who prepares a three-particle entangled state

$$|\varphi\rangle_{123} = \alpha_0|000\rangle_{123} + \alpha_1|111\rangle_{123}, \tag{2}$$

where α_0 is a real number, α_1 is a complex number and $|\alpha_0|^2 + |\alpha_1|^2 = 1$. This original state is unknown to both the sender Alice and the receiver Bob. Alice has safely received it from Victor and will use it as her input state. Alice wishes to teleport this state to Bob and then create a perfect copy of this state at her place with the assistance of Victor. For convenience, we use a shorthand $\oplus$ as plus module 2.

Assume that Alice and Bob share three entangled EPR pairs

$$|\psi\rangle_{A_tB_t} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_tB_t}, \quad t = 1, 2, 3, \tag{3}$$

where particles A_1, A_2, A_3 belong to Alice while particles B_1, B_2, B_3 belong to Bob. Hence, the initial state of the whole system can be written as

$$\begin{aligned}|\Psi\rangle &= |\varphi\rangle_{123} \otimes |\psi\rangle_{A_1B_1} \otimes |\psi\rangle_{A_2B_2} \otimes |\psi\rangle_{A_3B_3} \\ &= 2^{-\frac{3}{2}}\left(\sum_{j=0}^{1}\alpha_j|jjj\rangle_{123}\right)\left(\sum_{h_1=0}^{1}|h_1h_1\rangle_{A_1B_1}\right)\left(\sum_{h_2=0}^{1}|h_2h_2\rangle_{A_2B_2}\right)\left(\sum_{h_3=0}^{1}|h_3h_3\rangle_{A_3B_3}\right).\end{aligned} \tag{4}$$

Alice first performs the joint measurement on her particles $(1, A_1)$, $(2, A_2)$, $(3, A_3)$ under Bell basis

$$|\varphi_{nm}\rangle = \frac{1}{\sqrt{2}}\sum_{j=0}^{1}e^{\pi ijn}|j\rangle|j \oplus m\rangle, \quad n, m = 0, 1, \tag{5}$$

respectively. After these measurements, Alice sends the classical bits strings n_1m_1, n_2m_2, n_3m_3 to Bob through a classical channel, where $nm = 00(10, 01, 11)$ correspond to the Bell-state measurement result $|\varphi_{00}\rangle(|\varphi_{10}\rangle, |\varphi_{01}\rangle, |\varphi_{11}\rangle)$ respectively. Since

$$|jh\rangle = \frac{1}{\sqrt{2}}\sum_{n=0}^{1}e^{-\pi ijn}|\varphi_{n,h\oplus j}\rangle, \quad j, h = 0, 1, \tag{6}$$

(4) can be rewritten as

$$|\Psi\rangle = \frac{1}{2^3}\sum_{j,n_1,m_1,n_2,m_2,n_3,m_3=0}^{1}\left[e^{-\pi ij(n_1\oplus n_2\oplus n_3)}\alpha_j\prod_{t=1}^{3}|\varphi_{n_tm_t}\rangle_{tA_t}|j \oplus m_t\rangle_{B_t}\right]. \tag{7}$$

According to the received classical information, Bob operates a unitary transformation

$$(U_{n_1m_1})_{B_1} \otimes (U_{n_2m_2})_{B_2} \otimes (U_{n_3m_3})_{B_3} \tag{8}$$

on the remaining state

$$\sum_{j=0}^{1}e^{-\pi ij(n_1\oplus n_2\oplus n_3)}\alpha_j|j \oplus m_1\rangle_{B_1}|j \oplus m_2\rangle_{B_2}|j \oplus m_3\rangle_{B_3},$$

where

$$U_{nm} = \sum_{j=0}^{1}e^{\pi ijn}|j\rangle\langle j \oplus m|. \tag{9}$$

Then Bob gets $\sum_{j=0}^{1}\alpha_j|jjj\rangle_{B_1B_2B_3}$, which is the state Alice wants to transmit. The teleportation is successfully realized.

To create a perfect copy of the unknown state $|\varphi\rangle_{123}$, Alice needs the assistance of Victor. According to the projection postulate of quantum mechanics, if Alice applies projector

$|\varphi_{n_1m_1}\rangle_{1A_1}\langle\varphi_{n_1m_1}||\varphi_{n_2m_2}\rangle_{2A_2}\langle\varphi_{n_2m_2}||\varphi_{n_3m_3}\rangle_{3A_3}\langle\varphi_{n_3m_3}|$ onto the initial system $|\Psi\rangle$, the state of particles 1, A_1, 2, A_2, 3, A_3 will collapse into

$$|\varphi_{n_1m_1}\rangle_{1A_1}|\varphi_{n_2m_2}\rangle_{2A_2}|\varphi_{n_3m_3}\rangle_{3A_3}. \tag{10}$$

Alice applies a transformation

$$H_{A_3}C_{3A_3}C_{3A_2}(U_{n_3m_3})_{A_3}(U_{n_2m_2})_{A_2}(U_{n_1m_1})_{A_1} \tag{11}$$

on the state in (10), where U_{nm} is defined by (9), C_{3A_2} is a *CNOT* operation on particles 3 and A_2 (with A_2 the control qubit and 3 the target one), C_{3A_3} is a *CNOT* operation on particles 3 and A_3 (with A_3 the control qubit and 3 the target one), H_{A_3} is the Hadamard transformation on particle A_3. As we know,

$$(I\otimes U_{nm})|\varphi_{nm}\rangle = |\varphi_{00}\rangle = \frac{|00\rangle+|11\rangle}{\sqrt{2}}, \tag{12}$$

then Alice can get

$$\frac{1}{2}(|00\rangle+|11\rangle)_{1A_1}(|000\rangle+|111\rangle)_{23A_2}|0\rangle_{A_3} = \frac{1}{2}\sum_{h_1,j=0}^{1}|h_1jjh_1j\rangle_{123A_1A_2}|0\rangle_{A_3}. \tag{13}$$

Alice sends particles $(1,2,3)$ to Victor and keeps particles (A_1, A_2, A_3) in her possession.

Since Victor knows the original state $|\varphi\rangle_{123}$, he knows α_0 and α_1 completely. Victor performs a three-qubit projective measurement on the particles $(1,2,3)$ under the complete orthogonal basis

$$\begin{pmatrix}|\zeta_{000}\rangle\\|\zeta_{001}\rangle\\|\zeta_{010}\rangle\\|\zeta_{011}\rangle\\|\zeta_{100}\rangle\\|\zeta_{101}\rangle\\|\zeta_{110}\rangle\\|\zeta_{111}\rangle\end{pmatrix} = \begin{pmatrix}M & 0\\0 & M\end{pmatrix}^{\dagger}\begin{pmatrix}|000\rangle\\|001\rangle\\|010\rangle\\|011\rangle\\|100\rangle\\|101\rangle\\|110\rangle\\|111\rangle\end{pmatrix}, \tag{14}$$

where † denotes the conjugate transpose of a matrix and

$$M = \frac{1}{\sqrt{2}}\begin{pmatrix}\alpha_0 & \alpha_0 & \alpha_1 & \alpha_1\\ \alpha_1^* & \alpha_1^* & -\alpha_0 & -\alpha_0\\ \alpha_0 & -\alpha_0 & \alpha_1^* & -\alpha_1^*\\ \alpha_1 & -\alpha_1 & -\alpha_0 & \alpha_0\end{pmatrix}. \tag{15}$$

If the measurement result is $|\zeta_{k_1k_2k_3}\rangle$, Victor informs Alice of three bits $k_1k_2k_3$ via a classical channel. Since

$$|h_1jj\rangle_{123} = \frac{1}{\sqrt{2}}\sum_{h_2,h_3=0}^{1}e^{-\pi ij(h_2\oplus h_3)}\alpha_{j\oplus h_2}|\zeta_{h_1h_2h_3}\rangle_{123}, \quad h_1, j=0,1, \tag{16}$$

(13) can be rewritten as

$$\left(\frac{1}{\sqrt{2}}\right)^3 \sum_{h_1,j=0}^{1}\left[\sum_{h_2,h_3=0}^{1} e^{-\pi i j(h_2\oplus h_3)}\alpha_{j\oplus h_2}|\zeta_{h_1h_2h_3}\rangle_{123}\right]|h_1 j\rangle_{A_1A_2}|0\rangle_{A_3}. \tag{17}$$

According to the received classical bits $k_1k_2k_3$, Alice knows that the state of her qubits A_1, A_2, A_3 collapses into

$$\sum_{j=0}^{1} e^{-\pi i j(k_2\oplus k_3)}\alpha_{j\oplus k_2}|k_1 j\rangle_{A_1A_2}|0\rangle_{A_3}. \tag{18}$$

Equation (18) can be rewritten as

$$\sum_{j=0}^{1} e^{-\pi i (j\oplus k_2)(k_2\oplus k_3)}\alpha_{j}|k_1, j\oplus k_2\rangle_{A_1A_2}|0\rangle_{A_3}. \tag{19}$$

Then Alice operates a unitary transformation

$$X_{A_1}^{k_1}\otimes\left(\sum_{j=0}^{1} e^{\pi i (j\oplus k_2)(k_2\oplus k_3)}|j\rangle_{A_2}\langle j\oplus k_2|\right)\otimes I_{A_3} \tag{20}$$

on (19) and obtains

$$\sum_{j=0}^{1}\alpha_j|0\rangle_{A_1}|j\rangle_{A_2}|0\rangle_{A_3}. \tag{21}$$

Finally, Alice applies on her particles the control change gate

$$\prod_{j=1,3} C_{A_jA_2}, \tag{22}$$

where $C_{A_jA_2}$ is a *CNOT* operation (with A_2 the control qubit and A_j the target one), and obtains $\alpha_0|000\rangle_{A_1A_2A_3}+\alpha_1|111\rangle_{A_1A_2A_3}$. The clone of an unknown three-qubit entangled state is deterministically realized.

3 Deterministic Clone of an Unknown N-Qubit Entangled State with Assistance

We now turn to the general case of cloning N-qubit entangled state. Suppose the sender Alice has an input N-qubit entangled state represented by (1) from a state preparer Victor. The original state $|\varphi\rangle_{12\cdots N}$ is unknown to both the sender Alice and the receiver Bob. Alice wishes to help Bob to reestablish the original state and to create a perfect copy of the unknown state at her place with the assistance of Victor.

Assume that Alice and Bob share N entangled EPR pairs as the quantum channel, which are given by

$$|\psi\rangle_{A_tB_t}=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{A_tB_t},\quad t=1,\dots,N \tag{23}$$

where particles $A_1, \ldots, A_N$ belong to Alice while particles $B_1, \ldots, B_N$ belong to Bob. Hence, the initial state of the combined system can be written as

$$\begin{aligned}|\Psi\rangle &= |\varphi\rangle_{1\cdots N} \otimes |\psi\rangle_{A_1B_1} \cdots \otimes |\psi\rangle_{A_NB_N} \\ &= \left(\frac{1}{\sqrt{2}}\right)^N \left(\sum_{j=0}^{1} \alpha_j |j\cdots j\rangle_{1\cdots N}\right)\left(\sum_{h_1=0}^{1} |h_1h_1\rangle_{A_1B_1}\right)\cdots\left(\sum_{h_N=0}^{1} |h_Nh_N\rangle_{A_NB_N}\right). \end{aligned} \tag{24}$$

Alice first performs the joint measurement on her particles $(1, A_1), \ldots, (N, A_N)$ in Bell basis represented by (5) respectively. After these measurements, Alice sends the classical bit strings $n_1m_1, \ldots, n_Nm_N$ to Bob through a classical channel, where $nm = 00(10, 01, 11)$ correspond to the Bell-state measurement result $|\varphi_{00}\rangle(|\varphi_{10}\rangle, |\varphi_{01}\rangle, |\varphi_{11}\rangle)$, respectively. From (6), (24) can be rewritten as

$$|\Psi\rangle = \frac{1}{2^N} \sum_{j,n_1,m_1,\ldots,n_N,m_N=0}^{1} \left[e^{-\pi i j(n_1\oplus\cdots\oplus n_N)} \alpha_j \prod_{t=1}^{N} |\varphi_{n_tm_t}\rangle_{tA_t} |j \oplus m_t\rangle_{B_t} \right]. \tag{25}$$

According to the received classical information, Bob operates a unitary transformation

$$\prod_{t=1}^{N} (U_{n_tm_t})_{B_t}, \tag{26}$$

on the remaining state $\sum_{j=0}^{1}[e^{-\pi i j(n_1\oplus\cdots\oplus n_N)}\alpha_j \prod_{t=1}^{N} |j \oplus m_t\rangle_{B_t}]$, where U_{nm} is defined by (9). Then Bob gets $\sum_{j=0}^{1} \alpha_j |j\cdots j\rangle_{B_1\cdots B_N}$. The teleportation is successfully realized.

To create a perfect copy of the unknown state $|\varphi\rangle_{1\cdots N}$, Alice needs the assistance of Victor. According to the projection postulate of quantum mechanics, if Alice applies projector $|\varphi_{n_1m_1}\rangle_{1A_1}\langle\varphi_{n_1m_1}||\varphi_{n_2m_2}\rangle_{2A_2}\langle\varphi_{n_2m_2}|\cdots|\varphi_{n_Nm_N}\rangle_{NA_N}\langle\varphi_{n_Nm_N}|$ onto the initial system $|\Psi\rangle$, the state of particles $1, A_1, 2, A_2, \ldots, N, A_N$ will collapse into

$$|\varphi_{n_1m_1}\rangle_{1A_1}|\varphi_{n_2m_2}\rangle_{2A_2}\cdots|\varphi_{n_Nm_N}\rangle_{NA_N}. \tag{27}$$

Alice applies a transformation

$$H_{A_N} C_{NA_N} C_{NA_{N-1}} \prod_{j=1}^{N} (U_{n_jm_j})_{A_j} \tag{28}$$

on the state in (27), where U_{nm} is defined by (9), $C_{NA_{N-1}}$ is a *CNOT* operation on particles N and A_{N-1} (with A_{N-1} the control qubit and N the target one), C_{NA_N} is a *CNOT* operation on particles N and A_N (with A_N the control qubit and N the target one), H_{A_N} is the Hadamard transformation on particle A_N. Then Alice can get

$$\left(\frac{1}{\sqrt{2}}\right)^{N-1} \left[\prod_{t=1}^{N-2}\left(\sum_{h_t=0}^{1} |h_th_t\rangle_{tA_t}\right)\right]\left[\sum_{j=0}^{1} |jjj\rangle_{N-1,N,A_{N-1}}|0\rangle_{A_N}\right]. \tag{29}$$

Alice sends particles $(1, \ldots, N)$ to Victor and keeps particles $(A_1, \ldots, A_N)$ in her possession.

Since Victor knows the original state $|\varphi\rangle_{1\cdots N}$, he knows α_0 and α_1 completely. Victor performs an N-particle projective measurement on the particle $(1,\ldots,N)$ under the complete orthogonal basis

$$
\begin{aligned}
|\zeta_{\hat{l}00}\rangle_{12\cdots N} &= \frac{1}{\sqrt{2}}\prod_{j=1}^{N-2}|\rho_j(l)\rangle_j(\alpha_0|00\rangle+\alpha_1|01\rangle+\alpha_0|10\rangle+\alpha_1^*|11\rangle)_{N-1,N},\\
|\zeta_{\hat{l}01}\rangle_{12\cdots N} &= \frac{1}{\sqrt{2}}\prod_{j=1}^{N-2}|\rho_j(l)\rangle_j(\alpha_0|00\rangle+\alpha_1|01\rangle-\alpha_0|10\rangle-\alpha_1^*|11\rangle)_{N-1,N},\\
|\zeta_{\hat{l}10}\rangle_{12\cdots N} &= \frac{1}{\sqrt{2}}\prod_{j=1}^{N-2}|\rho_j(l)\rangle_j(\alpha_1^*|00\rangle-\alpha_0|01\rangle+\alpha_1|10\rangle-\alpha_0|11\rangle)_{N-1,N},\\
|\zeta_{\hat{l}11}\rangle_{12\cdots N} &= \frac{1}{\sqrt{2}}\prod_{j=1}^{N-2}|\rho_j(l)\rangle_j(\alpha_1^*|00\rangle-\alpha_0|01\rangle-\alpha_1|10\rangle+\alpha_0|11\rangle)_{N-1,N},
\end{aligned}
\tag{30}
$$

where $l=0,1,\ldots,2^{N-2}-1$. As we know, a decimal numeral l can be expressed as

$$l=\sum_{j=1}^{N-2}\rho_j(l)2^{j-1}=\rho_1(l)2^0+\cdots+\rho_{N-2}(l)2^{N-3}. \tag{31}$$

Define

$$\hat{l}=\rho_{N-2}(l)\cdots\rho_1(l). \tag{32}$$

If the measurement result is $|\zeta_{k_1\cdots k_{N-2}k_{N-1}k_N}\rangle$, Victor informs Alice of N bits $k_1\cdots k_{N-2}k_{N-1}k_N$ via a classical channel. Since

$$
\begin{aligned}
&|h_1\cdots h_{N-2}jj\rangle_{1\cdots N}\\
&\quad=\frac{1}{\sqrt{2}}\sum_{h_{N-1},h_N=0}^{1}e^{-\pi ij(h_{N-1}\oplus h_N)}\alpha_{j\oplus h_{N-1}}|\zeta_{h_1\cdots h_{N-2}h_{N-1}h_N}\rangle_{1\cdots N},\quad j=0,1,
\end{aligned}
\tag{33}
$$

(29) can be rewritten as

$$
\begin{aligned}
&\left(\frac{1}{\sqrt{2}}\right)^N\sum_{j=0}^{1}\sum_{h_1,\ldots,h_{N-2},h_{N-1},h_N=0}^{1}[e^{-\pi ij(h_{N-1}\oplus h_N)}\alpha_{j\oplus h_{N-1}}\\
&\quad\times|\zeta_{h_1\cdots h_{N-2}h_{N-1}h_N}\rangle_{1\cdots N}\otimes|h_1\cdots h_{N-2}\rangle_{A_1\cdots A_{N-2}}|j\rangle_{A_{N-1}}]|0\rangle_{A_N}.
\end{aligned}
\tag{34}
$$

According to the received classical bits $k_1\cdots k_{N-2}k_{N-1}k_N$, Alice knows that the state of her qubits $A_1\cdots A_N$ collapses into

$$\sum_{j=0}^{1}[e^{-\pi ij(k_{N-1}\oplus k_N)}\alpha_{j\oplus k_{N-1}}|k_1\cdots k_{N-2}\rangle_{A_1\cdots A_{N-2}}|j\rangle_{A_{N-1}}]|0\rangle_{A_N}. \tag{35}$$

Equation (35) can be rewritten as

$$\sum_{j=0}^{1} e^{-\pi i(j\oplus k_{N-1})(k_{N-1}\oplus k_N)}\alpha_j|k_1\cdots k_{N-2}\rangle_{A_1\cdots A_{N-2}}|j\oplus k_{N-1}\rangle_{A_{N-1}}|0\rangle_{A_N}. \tag{36}$$

Then Alice operates a unitary transformation

$$X_{A_1}^{k_1}\otimes\cdots\otimes X_{A_{N-2}}^{k_{N-2}}\otimes\left(\sum_{j=0}^{1} e^{\pi i(j\oplus k_{N-1})(k_{N-1}\oplus k_N)}|j\rangle_{A_{N-1}}\langle j\oplus k_{N-1}|\right)\otimes I_{A_N} \tag{37}$$

on (36) and obtains

$$\sum_{j=0}^{1}\alpha_j|0\cdots 0\rangle_{A_1\cdots A_{N-2}}|j\rangle_{A_{N-1}}|0\rangle_{A_N}. \tag{38}$$

Finally, Alice applies on her N qubits the control change gate

$$\prod_{j=1,\ldots,N-2,N} C_{A_jA_{N-1}}, \tag{39}$$

where $C_{A_jA_{N-1}}$ is a *CNOT* operation (with A_{N-1} the control qubit and A_j the target one), and obtains $\alpha_0|0\cdots 0\rangle_{A_1\cdots A_N}+\alpha_1|1\cdots 1\rangle_{A_1\cdots A_N}$. The clone of an unknown N-qubit entangled state is deterministically realized.

4 Conclusions

In conclusion, we have proposed an assisted-clone scheme which can produce perfect copies of an unknown $N(\geq 3)$-qubit entangled state in (1) when maximal entanglement is used as the quantum channel. The scheme includes two steps. In the first step, Alice performs Bell measurements and sends the classical bits to Bob via a classical channel. Based on Alice's classical information, Bob can reconstruct the original state on his particles. In the second step, a new transformation performed by Alice is constructed. Then we construct a set of mutually orthogonal basis. Under this basis, Victor performs an N-particle projective measurement on his particles which are sent by Alice. After having received Victor's measurement result, Alice can reestablish the original state. The probabilities of successful teleportation and clone of the original entangled state are both 1. Furthermore, the scheme can be extended to clone the unknown state via N non-maximally entangled states as the quantum channel. Under this assumption, the receiver can reestablish the unknown state with a certain probability. After the probabilistic teleportation [25], Alice still ends up with N Bell pairs which are the results of her measurements. Similar with the procedure as the quantum channel are maximally entangled, the copy of the input state can be produced at the sender's location deterministically.

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